430. Proposed by G. PAASWELL, New York City.

Revolve a circle about a chord (not a diameter). Select a system of rectilinear coördinates with this chord as one axis and the origin as the intersection of the chord and the circumference. Term this axis the z axis and pass a plane through the x (or y) axis. Find the area of this surface intercepted by this plane and the xz (or yz) plane.

MECHANICS.

346. Proposed by WILLIAM HOOVER, Columbus, Ohio.

Half the length of one of the equal parts of a uniform heavy string resting in equilibrium over a smooth horizontal indefinitely thin peg is cut off; determine the instantaneous change in the pressure on the peg.

347. Proposed by E. B. ESCOTT, Kansas City, Missouri.

A cord $ABCD$ is suspended from points A and D, which are 20 feet apart in horizontal distance. D is 4 feet lower than A. At B and C are suspended weights of 100 and 200 lbs. $AB = 8$ feet, $BC = 10$ feet, and $CD = 12$ feet. Find angles α, β, γ made by AB, BC, and CD, respectively, with the horizontal. Also find tensions T_1, T_2, and T_3 in AB, BC, and CD.

NUMBER THEORY.

265. Proposed by J. W. NICHOLSON, Louisiana State University.

If the roots of $x^4 - ax^2 + bx + c = 0$ are rational, prove that $4(a + yz) - 3(y + z)$ is a perfect square, y and z being any two roots of the equation.

245. Proposed by NORMAN ANNING, Chilliwack, B. C.

Show that $x^2 + y^2 = (a_1 a_2 \cdots a_n)^n$ has $4(n + 1)^m$ solutions in integers, in 2^{m+2} of which x and y are relatively prime, the a's being primes of the form $4k + 1$ and n a positive integer.

Note.—The proposer of this problem has changed it to read as above instead of the statement as previously published in the May, 1916, issue.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

469. Proposed by T. H. GRONWALL, New York City.

Show that the equation

$$f(x) \equiv 2ax^4 + (1 - b)x^3 + b(1 - b)x - 2ab = 0,$$

where $0 < b < 1$, $a > 0$ and $a^2 > b$, has only one positive root and that it lies between the roots of

$$g(x) \equiv x^2 - 2ax + b = 0.$$

SOLUTIONS BY D. R. CURTISS, Northwestern University.

I. By Descartes' Rule of Signs, $f(x)$ has only one positive root. Let x_1 be the smaller of the two roots of $g(x) = 0$, and x_2 be the larger. We shall have solved the problem if we establish the inequalities

$$f(x_1) < 0, \qquad f(x_2) > 0. \tag{1}$$

Let us now give b a constant value and let a vary from $\sqrt{b}$ to ∞. If X is either x_1 or x_2 we shall have $g(X) = X^2 - 2aX + b = 0$, and by differentiating with respect to a we obtain

$$\frac{dX}{da} = \frac{X}{X - a}. \tag{2}$$

We now have

$$\frac{df(X)}{da} = f'(X)\frac{dX}{da} + 2(X^4 - b) = \frac{f'(X)\cdot X + 2(X - a)(X^4 - b)}{X - a}$$